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PRESSURE DROP OF TWO-PHASE FLOW THROUGH CIRCULAR TUBES WITH IN-LINE STATIC MIXERS ACCOMPANIED BY CONDENSATION—SIMPLE STOCHASTIC MODELLING OF THE DATA

S. T. LIN* and L. T. FAN†
Kansas State University, Manhattan, KS 66506, U.S.A.

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NOMENCLATURE

A_r, heat transfer area between packets and wall;
C_{pm}, mean specific heat of the vapor–condensate mixture;
d, inside diameter of the condenser tube;
f, mean friction factor of the vapor–condensate mixture;
g_c, proportionality constant;
h_i, inside condensation heat transfer coefficient;
H_{fg}, latent heat;
H_{sp}, sensible heat;
k_m, mean thermal conductivity of the vapor–condensate mixture;
L, length of the condenser tube;
$\dot{M}$, mass flow rate of the condensing fluid;
Nu, Nusselt number of the vapor–condensate mixture, $h_i d/k_m$;
Pr_m, Prandtl number, $\mu_m C_{pm}/k_m$;
ΔP, pressure drop;
Re_l, Reynolds number of the condensate, $\dot{M}/\bar{\mu}_l d$;
ΔT, temperature difference, $(T_b - T_w)$;
$\bar{t}$, mean resident time;
u, velocity;
u_b, bulk velocity;
u_m, mean velocity of the vapor–condensate mixture;
$\dot{V}_l$, volumetric flow rate of the condensate.

Greek letters
θ, contact time;
$\bar{\rho}_l$, mean density of the condensate;
ρ_m, mean density of the vapor–condensate mixture;
δ, thickness of the packet;
$\bar{\mu}_l$, mean dynamic viscosity of the condensate;
μ_m, mean dynamic viscosity of the vapor–condensate mixture;
$\bar{\tau}$, mean period of the contact time;
ν_m, mean kinematic viscosity of the vapor–condensate mixture;
σ_i, instantaneous rate of momentum transfer;

$$\sigma_i(\theta) = \mu_m \left.\frac{\partial u}{\partial y}\right|_{y=0};$$

σ, mean rate of momentum transfer.

INTRODUCTION

THE AUGMENTATION of the rate of condensation heat transfer is of vital importance to many engineering systems and processes involving energy conversion, and chemical, physical and biological processing [1, 2]. Experimental evidence [3] indicates that the so-called in-line static mixers are highly effective in enhancing the rate of condensation heat transfer inside a circular tube while increasing the pressure drop across the tube.

Recently, a surface renewal model has been developed for the rate of condensation heat transfer in a tube with in-line static mixers [4]; however, a mechanistic model or a design correlation for the two phase pressure drop accompanied by condensation in such a tube is still lacking. The unique characteristics of in-line static mixers [4–7] suggest that the classical boundary-layer theory may not be suitable for modelling the condensation pressure drop (or momentum transfer) inside a tube with in-line static mixers. Development of an alternative approach, therefore, may be desirable. In 1956, Einstein and Li [8] introduced the concept of surface renewal to momentum transfer in a turbulent flow system. Meek and Baer [9, 10] extended the model by removing some of the assumptions. These successful applications of the surface renewal concept to the single-phase pressure drop suggested that the applicability of this concept to the two-phase pressure drop accompanied by condensation inside circular tubes with in-line static mixers be examined.

FORMULATION

Under the assumption that momentum is transferred from a packet of the vapor–condensate mixture to the inner tube surface by shear force during the contact between them, one has

$$\frac{\partial u}{\partial \theta} = \frac{\mu_m}{\rho_m}\frac{\partial^2 u}{\partial y^2} = \nu_m \frac{\partial^2 u}{\partial y^2}. \quad (1)$$

It is assumed that the velocity of the packet remains at a constant value of u_b beyond a certain characteristic length from the tube wall, δ, which is the size or thickness of the vapor–liquid packets. The solution of equation (1) subject to the initial and boundary conditions can be expressed as [11],

$$\frac{u - u_b}{-u_b} = 1 - \frac{y}{\delta} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\sin\frac{n\pi y}{\delta}\right)\exp(-\pi^2 n^2 \nu_m \theta/\delta^2). \quad (2)$$

The average rate of momentum transfer can be estimated simply by taking the sum of the individual contributions of the vapor–condensate packets of different ages, i.e.

$$\sigma = \int_0^{\infty} \sigma_i(\theta)\phi(\theta)\,d\theta, \quad (3)$$

where $\sigma_i(\theta)$ is the instantaneous rate of momentum transfer, and $\phi(\theta)$ is the contact time (or age) distribution function of the vapor–condensate packets. The rate of momentum transfer across a unit area of the tube surface for sufficiently large δ, $1/\tau$, or $1/\nu_m$ becomes [4],

* Department of Mechanical Engineering.
† Department of Chemical Engineering.

$$\sigma = u_m \rho_m \sqrt{\frac{\nu_m}{\tau}} \propto u_m \rho_m \sqrt{\frac{\nu_m}{t}}. \quad (4)$$

The Fanning friction factor for a homogeneous flow can be defined as [12]

$$f \equiv \frac{g_c(-\Delta P)d}{2u_m^2 \rho_m L} = \frac{2\sigma g_c}{u_m^2 \rho_m}. \quad (5)$$

From [4], one has

$$t \propto L/u_m$$

and

$$u_m \propto \frac{\dot{V}_l[(1-x)H_{fg} + H_{sp}]\dot{M}}{dk_m A_r \Delta T}.$$

Substitution of these expressions and equation (4) into equation (5) gives

$$f \propto \frac{d^2 \mu_m k_m \bar{\rho}_l \Delta T}{\rho_m \dot{M}^2[(1-x)H_{fg} + H_{sp}]}. \quad (6)$$

In dimensionless form, this expression becomes

$$f = \gamma \left\{ \frac{k_m}{\mu_m C_{pm}} \frac{d^2 \bar{\mu}_l^2}{\dot{M}^2} \frac{\mu_m^2}{\bar{\mu}_l^2} \frac{\bar{\rho}_l}{\rho_m} \frac{C_{pm}\Delta T}{(1-x)H_{fg} + H_{sp}} \right\}^{1/2}$$

or

$$[f] = \gamma [Pr_m]^{-1/2}[Re_l]^{-1} \left[\frac{\mu_m}{\bar{\mu}_l}\right]\left[\frac{\rho_m}{\bar{\rho}_l}\right]^{-1/2} \times \left[\frac{(1-x)H_{fg} + H_{sp}}{C_{pm}\Delta T}\right]^{-1/2}, \quad (7)$$

where γ is a proportional constant to be determined by experimental data.

EXPERIMENTAL

Thirty-two experimental data points previously obtained [3] were used here to verify the present model. The test facility employed in [3] included a vapor generator, superheater, vapor–liquid separator, after-condenser, liquid receiver and circulating pump. The facility also included two identical, horizontally mounted-in-parallel condensers, one of which had 44 static mixers. The ranges of operating conditions over which the data were obtained were:

(1) condensation temperature, 327.4 ~ 349.6 K;
(2) inlet superheat, 5.5 ~ 29.3 K;
(3) condensing pressure, 1.17×10^5 ~ 2.62×10^5 N/m^2;
(4) condensing fluid mass velocity, 33.3 ~ 102.7 Kg/m^2 s.

RESULTS AND DISCUSSION

Figure 1 plots, according to equation (7), the experimental data of condensation inside the tubes with in-line static mixers. A linear regression analysis of the data based on equation (7) yielded a value, $\gamma = 540.1$, with a correlation coefficient, r, of 0.92 and a standard deviation, $S_{y \cdot x}$, of 0.01. A test of linearity of the data by an analysis of variance involving the F-test [13], and a test of the null hypothesis, that the regression line passes through the origin by the t-test [14], indicate that such a linear relation adequately correlates the data.

From the result of our previous work [4], one has

$$[Nu] = 1.78[Pr_m][Re_l]\left[\frac{\mu_l}{\mu_m}\right] \times \left[\frac{d}{L}\right]\left[\frac{\rho_m}{\bar{\rho}_l}\right]^{1/2}\left[\frac{(1-x)H_{fg} + H_{sp}}{C_{pm}\Delta T}\right]^{1/2}. \quad (8)$$

Solving equation (8) for $[(1-x)H_{fg} + H_{sp}/C_{pm}\Delta T]$ and substituting the resultant expression into equation (7), one has

$$[Nu] = \beta [Pr_m]^{1/2}\left[\frac{d}{L}\right][f]^{-1}, \quad (9)$$

where β is the constant. Equation (9) expresses the analogy between the heat transfer and fluid friction for condensation of vapor inside a circular tube with in-line static mixers. Figure 2 plots the experimental data according to equation (9). Again, the F-test [13] and the t-test [14] of the analysis of the data indicate that equation (9) adequately correlates the data with a value $\beta = 957.6$. Therefore, the forced conden-

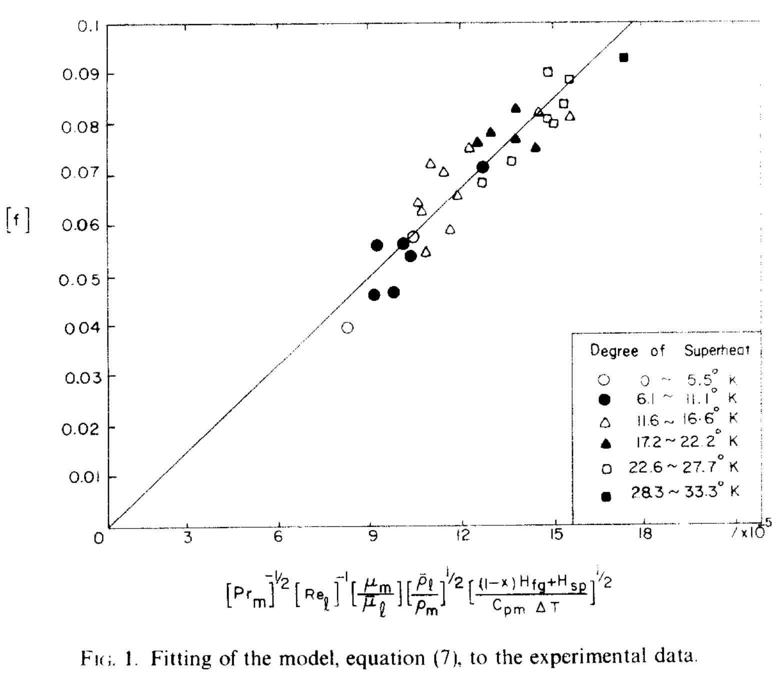

FIG. 1. Fitting of the model, equation (7), to the experimental data.

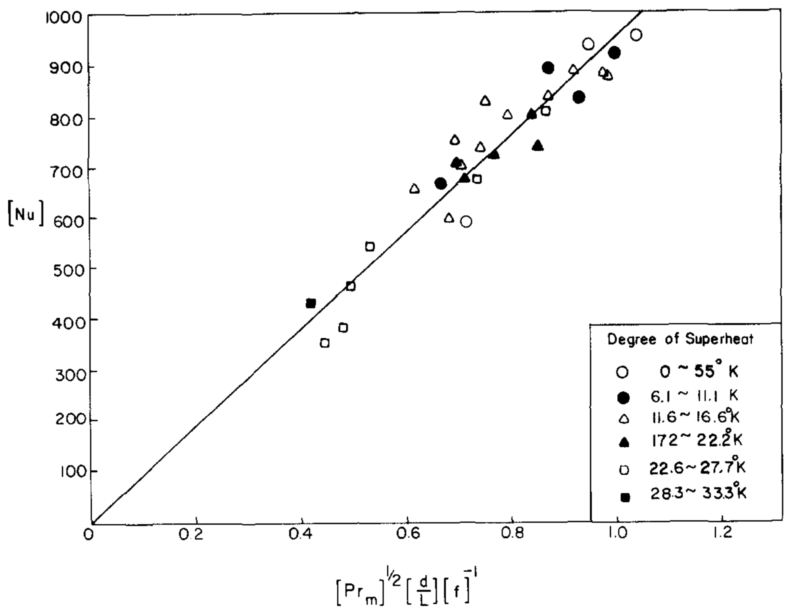

FIG. 2. Fitting of the model, equation (9), to the experimental data.

sation heat transfer coefficient inside a circular tube with in-line static mixers can be determined by making measurements of the fluid friction and vice versa.

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